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A Proposal of Telecloning for a Three-Particle Entangled W State

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Abstract A $1 \to 2$ telecloning solution for an arbitrary three-particle entangled W state is proposed in which two four-particle entangled states are used as quantum channels. It is proposed that the three-particle W state can be telecloned based on the quantum teleportation and the local copying of entanglement, and the fidelity of each clone depends on the input state. This scheme can be generalized into the case of $1 \to N$ $(N > 2)$ telecloning of an arbitrary three-particle W state. Furthermore, another scheme for $1 \to N$ $(N \geq 2)$ telecloning of an arbitrary n-particle $(n \geq 4)$ W state is proposed, the multi-bit controlled-NOT (CNOT) gates and additional particles are needed in this case.

Keywords Quantum information · W state · Quantum telecloning · Two-bit CNOT gate

1 Introduction

In quantum information processing, there are many restrictions and limitations because of the principles of quantum physics. A well-known fact is the so-called quantum no-cloning theorem raised by Wotters and Zurek [1] and Dieks [2], which asserts that an unknown quantum state cannot be perfectly cloned. Although exact cloning is not possible, much attention has been focused on the approximate cloning [3, 4]. As a result, various cloning machines have been proposed which operate either in the universal optimal cloning [5, 6] or the probabilistic cloning [7, 8]. Subsequently, Murao et al. [9] have given a scheme of quantum telecloning combing quantum teleportation and optimal universal quantum cloning, which can accomplish the task of broadcasting information encoded in a qubit from one sender to M spatially separated receivers. Then, many kinds of proposals for telecloning single-qubit

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state have been given theoretical and experimental as well [10–16]. With the great advances in quantum cryptography, quantum remote control and distributed quantum computing, the effective proposals for transmission, manipulation and storage of quantum information are required, which can be achieved by utilizing the capability of creating and manipulating of nonlocal multipartite quantum entanglement. So, there has been a steadily growing interest in telecloning and telebroadcasting for quantum entangled states [17–20]. Compared with the scheme in Ref. [19], the proposal in Ref. [20] not only weakened the requirements for the quantum channel, but also increased the fidelity of the copy.

It has been shown that for systems of three or more particles there are several inequivalent classes of entangled sates such as Greenberger-Home-Zeilinger (GHZ) states and W states [21]. In the past few years, many researchers have extensively studied GHZ states due to their important role in the quantum information processing and communication [22–24]. It should be noted that schemes to realize $1 \to N$ $(N \geq 2)$ telecloning of n-particle $(n \geq 3)$ GHZ state have been presented in Refs. [19, 20]. Recently, much public attention is attracted to study W states [25–27]. This is a consequence of their special properties: their entanglement is not only maximally persistent and robust under particle loss, but also they are immune against global dephasing and bit flip noise. So far, there are several experimental schemes demonstrated for the implementation of phase-covariant cloning machine to clone W states [28, 29]. In this paper we propose a theoretical scheme for telecloning of an unknown three-particle W state. In our scheme, an unknown W state can be telecloned, sharing the same entangled states as in Ref. [19], once one has at disposal the following ingredients: (i) two Bell-state measurements; (ii) a Hadamard transformation and a projective measurement at the sender's side; (iii) an additional particle, a two-bit CNOT gate as well as unitary transformations at each of receivers' locations.

Our paper is organized as follows. In Sect. 2, we present our scheme to implement $1 \to 2$ telecloning of three-particle W state that combines quantum teleportation and local copying of entanglement in detail. In Sect. 3,we generalize this scheme into the case of telecloning of an arbitrary n-particle $(n \geq 4)$ W state. Open questions and conclusions are presented in Sect. 4.

2 1 → 2 Telecloning of a Three-Particle W State

Let us consider the following situation. Alice has at her side an unknown three-particle W entangled state of the form

$$|\psi_w\rangle_{123} = (\alpha|001\rangle + \beta|010\rangle + \gamma|100\rangle)_{123}, \quad |\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1, \tag{1}$$

and she wants to transmit identical copies of it to 2 spatially separated receivers Bob, Charlie. Suppose that they share two maximally four-particle entangled states as quantum channels described by:

$$|\psi\rangle_C = \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_1}|\phi_0\rangle_{A_1B_1C_1} + |1\rangle_{P_1}|\phi_1\rangle_{A_1B_1C_1})\right] \otimes \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_2}|\phi_0\rangle_{A_2B_2C_2} + |1\rangle_{P_2}|\phi_1\rangle_{A_2B_2C_2})\right], \quad \text{where} \tag{2}$$

$$|\phi_0\rangle_{A_iB_iC_i} = \sqrt{\frac{2}{3}}|0\rangle|0\rangle|0\rangle + \sqrt{\frac{1}{6}}(|1\rangle|0\rangle|1\rangle + |1\rangle|1\rangle|0\rangle) \quad (i = 1, 2), \tag{3}$$

$$|\phi_1\rangle_{A_iB_iC_i} = \sqrt{\frac{2}{3}}|1\rangle|1\rangle|1\rangle + \sqrt{\frac{1}{6}}(|0\rangle|1\rangle|0\rangle + |0\rangle|0\rangle|1\rangle) \quad (i = 1, 2). \tag{4}$$

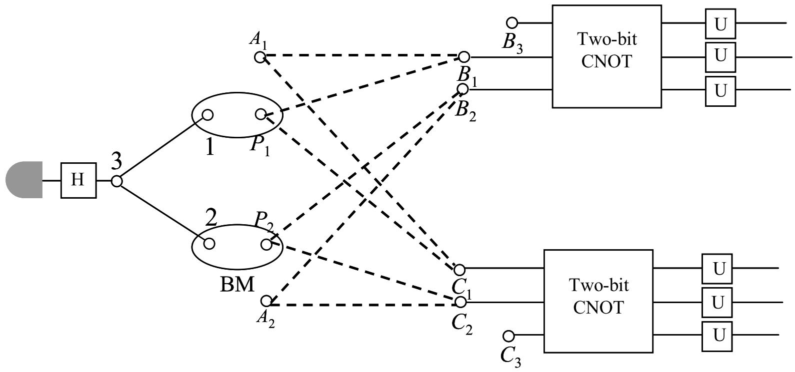

Fig. 1 Schematic of telecloning for the three-particle W state to two users. This protocol is obtained by the combing teleportation and the optimal universal cloning. The quantum channels used are two maximally entangled four-particle states. The *dotted lines* indicate the entanglement of the quantum channels. The sender has to perform locally two Bell-state measurements, a Hadamard transformation, and a projective measurement, announce the outcomes to the receivers, who perform local recovery unitary transformations with the assistance of auxiliary particles and two-bit CNOT gates obtaining the information of the initial unknown state

Here, particles P_1, P_2 represent qubits held by Alice, which we shall refer to as the "port" qubits; A_1, A_2 denote auxiliary particles that can be placed on Alice's side for convenience; Bob is in possession of particles B_1, B_2 and Charlie is in possession of C_1, C_2.

The initial state of the system composed of the original state and the quantum channels is $|\psi_T\rangle = |\psi_w\rangle_{123}|\psi\rangle_C$. The telecloning of $|\psi_w\rangle_{123}$ can be accomplished by the following procedure (see Fig. 1).

(i) Alice performs two Bell measurements (BM) on particles 1, P_1 and 2, P_2 respectively. There are sixteen possible outcomes:

$$\begin{aligned}{}_{1P_1}\langle\phi^{\pm}|_{2P_2}\langle\phi^{\pm}|\psi_T\rangle = \frac{1}{4}(\alpha\,|1\rangle_3\,|\phi_0\rangle_{A_1B_1C_1}\,|\phi_0\rangle_{A_2B_2C_2} \pm^2 \beta\,|0\rangle_3\,|\phi_0\rangle_{A_1B_1C_1}\,|\phi_1\rangle_{A_2B_2C_2}\\ \pm^1 \gamma|0\rangle_3|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2}),\end{aligned} \tag{5}$$

$$\begin{aligned}{}_{1P_1}\langle\phi^{\pm}|_{2P_2}\langle\psi^{\pm}|\psi_T\rangle = \frac{1}{4}(\alpha\,|1\rangle_3\,|\phi_0\rangle_{A_1B_1C_1}\,|\phi_1\rangle_{A_2B_2C_2} \pm^2 \beta\,|0\rangle_3\,|\phi_0\rangle_{A_1B_1C_1}\,|\phi_0\rangle_{A_2B_2C_2}\\ \pm^1 \gamma|0\rangle_3|\phi_1\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2}),\end{aligned} \tag{6}$$

$$\begin{aligned}{}_{1P_1}\langle\psi^{\pm}|_{2P_2}\langle\phi^{\pm}|\psi_T\rangle = \frac{1}{4}(\alpha\,|1\rangle_3\,|\phi_1\rangle_{A_1B_1C_1}\,|\phi_0\rangle_{A_2B_2C_2} \pm^2 \beta\,|0\rangle_3\,|\phi_1\rangle_{A_1B_1C_1}\,|\phi_1\rangle_{A_2B_2C_2}\\ \pm^1 \gamma|0\rangle_3|\phi_0\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2}),\end{aligned} \tag{7}$$

$$\begin{aligned}{}_{1P_1}\langle\psi^{\pm}|_{2P_2}\langle\psi^{\pm}|\psi_T\rangle = \frac{1}{4}(\alpha\,|1\rangle_3\,|\phi_1\rangle_{A_1B_1C_1}\,|\phi_1\rangle_{A_2B_2C_2} \pm^2 \beta\,|0\rangle_3\,|\phi_1\rangle_{A_1B_1C_1}\,|\phi_0\rangle_{A_2B_2C_2}\\ \pm^1 \gamma|0\rangle_3|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2}),\end{aligned} \tag{8}$$

where $|\phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are Bell states, and where $\pm^1$ and $\pm^2$ correspond to the superscripts for the Bell-state composed of particles $(1, P_1)$ and $(2, P_2)$, respectively. For instance, if Alice measurement results are $|\psi^+\rangle_{1P_1}$ and $|\psi^-\rangle_{2P_2}$, i.e., the corresponding superscripts are "+" and "−", the state as shown by (8) will collapse into:

$$|\psi\rangle_{3A_1B_1C_1A_2B_2C_2} = \frac{1}{4}(\alpha|1\rangle_3|\phi_1\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2} - \beta|0\rangle_3|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2} + \gamma|0\rangle_3|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2}). \quad (9)$$

(ii) Then Alice carries out a Hadamard operation on particle 3. For example, the Hadamard operation

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle); \qquad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \quad (10)$$

will transform (9) into:

$$\frac{1}{4\sqrt{2}}[(\alpha|\phi_1\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2} - \beta|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2} + \gamma|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2})|0\rangle_3 + (-\alpha|\phi_1\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2} - \beta|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2} + \gamma|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2})|1\rangle_3]. \quad (11)$$

Alice measures particle 3. If the result is $|1\rangle_3$, the state will be projected to

$$\frac{1}{4\sqrt{2}}(-\alpha|\phi_1\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2} - \beta|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2} + \gamma|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2}). \quad (12)$$

Alice publicly broadcasts all the outcomes to the two receivers by classical information channel.

(iii) The receivers recover the correct state with their particles by performing unitary transformations according to the classical information they received. This procedure can be divided into three steps.

Firstly, Bob and Charlie perform proper Pauli-X matrix operation $U(\sigma_x)$ based on the Bell measurements.

Secondly, the two receivers introduce auxiliary particles B_3 and C_3 with theirs initial state $|0\rangle$ separately. Then, the two receives employ a two-bit CNOT gate [30] which acts on particles (B_1, B_2, B_3) and (C_1, C_2, C_3), respectively. We must stress that here $B_3(C_3)$ is target bit which can flip if and only if control bits B_1, $B_2(C_1, C_2)$ are both in the state $|0\rangle$.

Finally, in order to acquire the approximate copies of initial state, the two receivers need perform appropriate Pauli-Z matrix operation $U(\sigma_z)$ depending on the different signs before α, β, γ caused by the Bell measurements and the Hadamard transformation.

Specifically, if $|\psi^+\rangle_{1P_1}, |\psi^-\rangle_{2P_2}$ and $|1\rangle_3$ are achieved (see (12)), on the basis of the three steps described above, to begin with, Bob can carry out single-qubit operation $(\sigma_x)_{B_1}(\sigma_x)_{B_2}$, and in this case, (12) becomes:

$$\frac{1}{4\sqrt{2}}(-\alpha|\phi_0\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2} - \beta|\phi_0\rangle_{A_1B_1C_1}|\phi_1\rangle_{A_2B_2C_2} + \gamma|\phi_1\rangle_{A_1B_1C_1}|\phi_0\rangle_{A_2B_2C_2}). \quad (13)$$

Next, he performs single-qubit transformations $(\sigma_z)_{B_3}(\sigma_z)_{B_2}$ according to the sings "−, −, +" after introducing an auxiliary particle B_3 and letting his particles B_1, B_2, B_3 enter a two-bit CNOT gate. The works Charlie needs do are similar to Bob. So far, the telecloning of three-particle W state has been accomplished. The receivers obtain the density operator:

$$
\begin{aligned}
\rho_{B_1B_2B_3} &= \rho_{C_1C_2C_3} \\
&= \left(\frac{24|\alpha|^2+1}{36}+\frac{|\beta|^2+|\gamma|^2}{9}\right)|001\rangle\langle 001| + \left(\frac{24|\beta|^2+1}{36}+\frac{|\alpha|^2}{9}\right)|010\rangle\langle 010| \\
&\quad + \left(\frac{24|\gamma|^2+1}{36}+\frac{|\alpha|^2}{9}\right)|100\rangle\langle 100| + \frac{5}{9}\alpha\beta^*|001\rangle\langle 010| + \frac{5}{9}\alpha\gamma^*|001\rangle\langle 100| \\
&\quad + \frac{5}{9}\beta\alpha^*|010\rangle\langle 001| + \frac{4}{9}\beta\gamma^*|010\rangle\langle 100| + \frac{1}{9}\alpha\gamma^*|010\rangle\langle 110| \\
&\quad + \frac{5}{9}\gamma\alpha^*|100\rangle\langle 001| + \frac{4}{9}\gamma\beta^*|100\rangle\langle 010| + \frac{1}{9}\alpha\beta^*|100\rangle\langle 110| \\
&\quad + \frac{1}{9}\gamma\alpha^*|110\rangle\langle 010| + \frac{1}{9}\beta\alpha^*|110\rangle\langle 100| + \left(\frac{1}{36}+\frac{|\beta|^2+|\gamma|^2}{9}\right)|110\rangle\langle 110|.
\end{aligned}
\tag{14}
$$

According to the definition of fidelity and (14), the receivers obtain

$$
F_B = {}_{123}\langle\psi_w|\rho_{B_1B_2B_3}|\psi_w\rangle_{123} = F_C = {}_{123}\langle\psi_w|\rho_{C_1C_2C_3}|\psi_w\rangle_{123} = \frac{25-16|\beta|^2|\gamma|^2}{36}. \tag{15}
$$

Noticing $|\alpha|^2+|\beta|^2+|\gamma|^2=1$, $0\le|\beta|^2\le 1$, F is in the range of $21/36\le F\le 25/36$; the left equality holds if $|\beta|^2=|\gamma|^2=1/2$, $|\alpha|^2=0$ and the right equality holds if $|\beta|^2=0$ or $|\gamma|^2=0$. In Fig. 2 we show the fidelity as a function of the parameter $|\beta|^2$ for different values

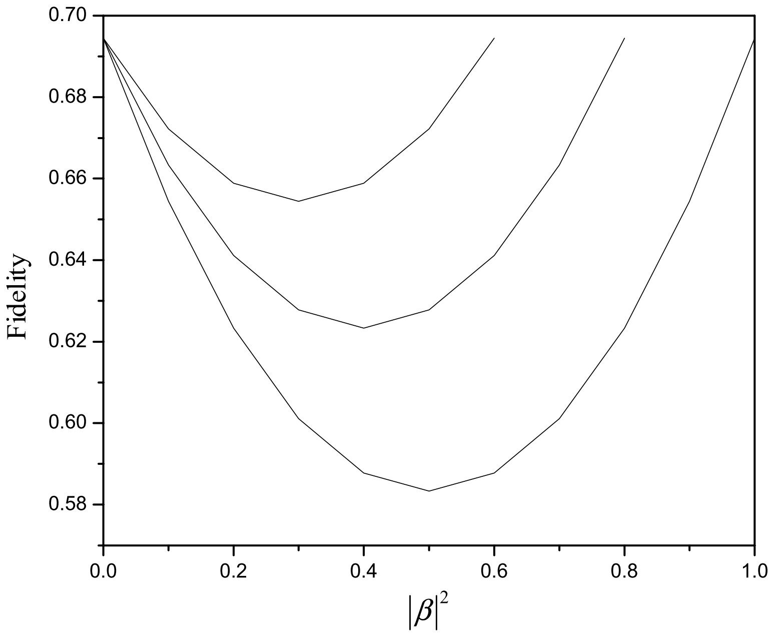

Fig. 2 Fidelity of the telecloning as a function of the parameter $|\beta|^2$ for different values of $|\alpha|^2$. From *bottom* to *top* the *curves* are plotted for $|\alpha|^2=0$; 0.2; 0.4

of $|\alpha|^2$, what we find is that the least value of fidelity increases as the value of $|\alpha|^2$ increases, although each of curves has the same greatest value for all values of $|\alpha|^2$. Obviously, F is dependent on the input state, so the proposal we have described above is not a universal process (see Fig. 2).

Certainly, we could use the same principle to derive another telecloning scenario, in which three maximally four-particle entangled states are used as quantum channels. In this case, for sender what she needs do are three Bell measurements; for each of the receivers, all he needs are local unitary transformations, i.e., the logic gate and the accessory qubit are not necessary. However, this process will consume much resource of quantum entanglement and classical information; most of all, the fidelity of the copies is less than the value acquired in the first scheme. Here, we give the expression of the fidelity of the second proposal:

$$\begin{aligned} F'_B &= {}_{123}\langle\psi_w|\rho_{B_1B_2B_3}|\psi_w\rangle_{123} = F'_C = {}_{123}\langle\psi_w|\rho_{C_1C_2C_3}|\psi_w\rangle_{123} \\ &= \frac{125 - 80(|\alpha|^2|\beta|^2 + |\beta|^2|\gamma|^2 + |\alpha|^2|\gamma|^2)}{216}. \end{aligned} \tag{16}$$

Conclusion shows that F is in the range of $[125 - (80/3)]/216 \le F \le 125/216$, the left equality holds if $|\alpha|^2 = |\beta|^2 = |\gamma|^2 = 1/3$, and the right equality holds if one of the three coefficients equal to 1, and the other two coefficients equal to 0. According to the comparison of the two schemes, we can say that with the help of an auxiliary qubit and a logic gate, we used fewer nonlocal quantum entanglement resources as quantum channels in the first scheme, i.e., we have realized the intention of saving expensive nonlocal entanglement resources, and, as noted, this intention has become a hot topic recently [20, 31–33]; on the other hand, carrying out a Hadamard transformation on particle 3 of the original state instead of applying the optimal universal cloning can derive a higher fidelity.

The proposal can be generalized into the case of $1 \to N$ telecloning of an arbitrary three-particle W state. We choose that the sender and the receivers share two $2N$-particle entangled states as the quantum channels described by:

$$\begin{aligned} |\psi\rangle_{P_1A_1R_1P_2A_2R_2} &= \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_1}|\phi_0\rangle_{A_1R_1} + |1\rangle_{P_1}|\phi_1\rangle_{A_1R_1})\right] \\ &\quad \otimes \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_2}|\phi_0\rangle_{A_2R_2} + |1\rangle_{P_2}|\phi_1\rangle_{A_2R_2})\right], \end{aligned} \tag{17}$$

where

$$|\phi_0\rangle_{A_iR_i} = \sum_{j=0}^{N-1}\alpha_j|\{0, N-1-j\}, \{1, j\}\rangle_{A_i} \otimes |\{0, N-j\}, \{1, j\}\rangle_{R_i} \quad (i = 1, 2), \tag{18}$$

$$|\phi_1\rangle_{A_iR_i} = \sum_{j=0}^{N-1}\alpha_j|\{0, j\}, \{1, N-1-j\}\rangle_{A_i} \otimes |\{0, j\}, \{1, N-j\}\rangle_{R_i} \quad (i = 1, 2), \tag{19}$$

$\alpha_j = \sqrt{\frac{2(N-j)}{N(N+1)}}$, P_1 and P_2 represent qubits held by Alice which functions as input "ports", A_i denotes $N-1$ auxiliary particles, R_i denotes the N qubits which are held by the spatially separated receivers $B_1, B_2, \ldots, B_N$. Calculation shows that with assistance of additional qubits and two-bit CNOT gates at the receivers' locations, by means of local quantum measurements, a Hadamard transformation, classical information, local unitary transformations, the task of $1 \to N$ telecloning of an arbitrary three-particle W state can be accomplished.

3 Telecloning of an *n*-Particle W State

We start our work with $1 \to 2$ quantum telecloning of an unknown n-particle ($n \geq 4$) W state. We suppose that an unknown n-particle W state Alice wants to teleclone to Bob and Charlie has the form:

$$\begin{aligned}|\psi\rangle_{1,2,\ldots,n} &= (d_1\,|100\cdots 0\rangle + d_2\,|010\cdots 0\rangle + \cdots + d_n\,|000\cdots 1\rangle)_{1,2,\ldots,n} \\ &= \sum_{k=1}^{n} d_k \prod_{i=1}^{n} |\delta_{ki}\rangle_i,\end{aligned} \tag{20}$$

where

$$\sum_{k=1}^{n} |d_k|^2 = 1, \quad d_k \geq 0 \quad (k = 1, 2, \ldots, n), \qquad \delta_{ki} = \begin{cases} 0, & k \neq i, \\ 1, & k = i. \end{cases}$$

In this case, the sender and the receivers share $n-1$ four-particle entangled states as the quantum channels given by:

$$|\psi\rangle_{P_1A_1B_1C_1\ldots P_{(n-1)}A_{(n-1)}B_{(n-1)}C_{(n-1)}} = \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_1}|\phi_0\rangle_{A_1B_1C_1} + |1\rangle_{P_1}|\phi_1\rangle_{A_1B_1C_1})\right]^{\otimes(n-1)}, \tag{21}$$

where particles $P_1, P_2, \ldots, P_{n-1}$ are held by the sender, $A_1, A_2, \ldots, A_{n-1}$ denote auxiliary particles, whereas particles $B_1, B_2, \ldots, B_{n-1}$ and $C_1, C_2, \ldots, C_{n-1}$ are held by the spatially separated receiver Bob, Charlie, respectively.

Alice performs Bell measurements on particles (i, P_i) $(i = 1, 2, \ldots, n-1)$ separately, after a Hadamard transformation, measures particle n. Then she sends to Bob and Charlie the results of both measurements. Bob performs unitary transformations $U(\sigma_x)$ based on the outcomes of Bell measurements; subsequently, he introduces the additional particle B_N with its initial state $|0\rangle$, then make his own particles $(B_1, B_2, \ldots, B_{n-1}, B_n)$ through multi-bit CNOT gate. Here, $(B_1, B_2, \ldots, B_{n-1})$ are control bits, B_n is a target bit. It might also be noted that if and only if control bits are all in the state $|0\rangle$, the target bit flip. Finally, Bob performs appropriate unitary transformation $U(\sigma_z)$ corresponding to the sings before $(d_1, d_2, \ldots, d_n)$. By this time, the $1 \to 2$ quantum telecloning of an unknown n-particle ($n \geq 4$) W state has been accomplished. The other receiver Charlie can simultaneously obtain approximate copy on the basis of the steps mentioned above.

Then we demonstrate that the present scheme can be generalized to teleclone an unknown n-particle ($n \geq 4$) W state from a sender to N spatially separated receivers. We choose that the sender and the receivers share $n-1$ $2N$-particle entangled sates as the quantum channels given by:

$$|\psi\rangle_{P_1A_1R_1\ldots P_{n-1}A_{n-1}R_{n-1}} = \left[\frac{1}{\sqrt{2}}(|0\rangle_{P_1}|\phi_0\rangle_{A_1R_1} + |1\rangle_{P_1}|\phi_1\rangle_{A_1R_1})\right]^{\otimes n-1}, \tag{22}$$

where the meaning of particles P_i, A_i, and R_i $(i = 1, 2, \ldots, n-1)$ has been presented in Sect. 2, $|\phi_0\rangle_{A_iR_i}$, $|\phi_1\rangle_{A_iR_i}$ have the same expressions as (17) and (18). Each receiver can acquire the approximate copy of the initial state assisted by an additional particle and a multi-bit CNOT gate, through the same processes described above.

4 Conclusions

One scheme for telecloning a three-particle W sate to 2 distant users has been proposed. We choose two four-particle entangled states shared by the sender and the receivers as the quantum channels, after the sender's two Bell measurements, a Hadamard operation and a projective measurement, classical communication; as well as each receiver's local unitary transformations with the help of an auxiliary particle and a two-bit CNOT gate, the spatially separated receivers get one approximate copy of the original unknown state. Then we generalize the scheme into the case of $1 \to N$ $(N > 2)$ telecloning of an arbitrary three-particle W state. If each receiver has an auxiliary particle and a multi-bit CNOT gate, the sender and the receivers share two $2N$-particle entangled states as quantum channels, M spatially separated receivers get one approximate copy of the original unknown state through local quantum measurements, a Hadamard operation, classical information and local unitary transformations. In the mean time, we present a scheme to implement $1 \to 2$ telecloning for an arbitrary n-particle $(n \geq 4)$ W state, then generalize it into the case of $1\to N(N > 2)$ telecloning of an arbitrary n-particle W state.

Since all these operations are within the reach of current technology [5, 30], our scheme for $1 \to N$ telecloning is experimentally feasible, this scheme is helpful for the study of quantum communication and nonlocal multipartite distribution of quantum information.

There are some open questions about nonlocal multipartite distribution and manipulation of quantum information. For example, one open question is how to implement the task of universal telecloning of multiparticle W state, we should seek for an adequate solution to the problem.

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